Symmetry-Protected Topological Order

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1. Motivation. Recapitulation of concepts from field theory and differential geometry
   - 1.1. Outline of topological concepts in quantum information science, electronics and quantum materials to be discussed in this lecture course
   - 1.2. Basic ideas about differential geometry and Riemannian manifolds.
   - 1.3. Path integral formulation of quantum mechanics and quantum field theory

   Exercise: Bosonic coherent states

2. Topological terms and path integral: 0+1 space-time dimension
   - 2.1. Spin coherent states and path integral for a single spin
   - 2.1. Summary of types of topological terms

   Exercise: Path integral for a particle on a flux-threaded ring

3. Topological terms and path integral: D+1 space-time dimensions
   - 3.1. D = 1: Antiferromagnetic spin chain, Haldane conjecture
   - 3.2. Spin-1 chain with open boundary conditions, AKLT solution. Outlook on Matrix-Product states and DMRG
   - 3.3. Definition of symmetry protected topological order in D _ 1

4. Kramers Wannier duality and Wegner's Z2 gauge theory
   - 4.1. 1D transverse field Ising model – order and disorder operators. Kramers Wannier (self-) duality
   - 4.2. 2D transverse field Ising model – order and disorder operators, Wegner's Z2 gauge theory. De-confinement
   - 4.3. Z2 gauge theory with matter and duality to the toric code

   Exercise: 1D clock models, order and disorder operators, lattice parafermions

5. The toric code
   - 5.1. Soluble limit of the toric code: deconfining ground state as in Wegner's Z2 gauge theory
   - 5.2. e, m and ε excitations in the toric code. Exchange statistics
   - 5.3. Definitions of topological order

   Exercise: ZN toric code

6. Fractionalized and orthogonal Fermi liquids
   - 6.1. Luttinger-Oshikawa theorem
   - 6.2. Fractionalized Fermi liquids and orthogonal Fermi liquids
   - 6.3. Fermi surface reconstruction without symmetry breaking

7. Kitaev's honeycomb model
   - 7.1. Exact solution, gapped and gapless phases.
   - 7.2. Application of magnetic field
   - 7.3. Discussion of candidate materials ($RuCl_3$ and iridates)

   – Exercise: Topology of Bloch states in the gapped phase

8. (Non-Abelian) Anyons and 16-fold way
   - 8.1. General concepts: Fusion rules, quantum dimensions, exchange matrices, braiding

8.2. Illustrative examples: Ising and Fibonacci anyons
8.3. Kitaev 16-fold way classification
8.4. Outlook to topological quantum computation