Random Matrix Theory: Wigner-Dyson statistics and beyond

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1. Application of random matrices in physics
2. Invariant and non-invariant random matrix ensembles
   2.1. Probability distribution of invariant and Gaussian-non-invariant ensemble
   2.2. Strong and weak confinement
   2.3. Special cases of non-invariant ensembles: Rosenzweig-Porter and Power-law banded random matrices
3. Joint probability distribution of eigenvalues and eigenvectors of Wigner-Dyson ensemble and level repulsion
   3.1. The Jacobian and Vandermond determinant
   3.2. Level repulsion (poor men derivation)
4. Dyson symmetry classes and their extension
   4.1 Time-reversal symmetry and the Dyson symmetry classes
   4.2. Particle-hole symmetry and the 10-fold way Cartan symmetry classes
5. Level statistics of Wigner-Dyson RMT as 1D gas of quantum and classical particles
   5.1. Classical plasma of logarithmically-interacting particles
   5.2. Calogero-Sutherland models of interacting fermions
6. Wigner semi-circle and the probability of hole creation in a 1D log-interacting plasma
   6.1. Wigner semi-circle from the solution of integral equation for equilibrium profile of log-interacting 1D plasma
   6.2. Probability of a hole in log-interacting plasma and the tail of the level spacing distribution
   6.3. Hole production at weak confinement
7. Level compressibility, normalization sum rule and normalization anomaly
   7.1. Dos correlation function
   7.2. The level number variance
   7.3. The sum rule and the normalization anomaly
8. Solution of invariant ensembles by orthogonal polynomials
   8.1. Orthogonal polynomials and there-term recursive relation
   8.2. Expression for DoS correlation function in terms of orthogonal polynomials
   8.3. The Wigner-Dyson correlation kernel and higher-order correlation functions
9. WKB quasi-classical approximation for orthogonal polynomials and the one-dimensional Wigner crystal of energy levels
10. Logarithmic confinement and DoS correlation function
    10.1. Dos at logarithmic confinement
    10.2. Unfolding and the DoS correlation function: emergence of a “ghost correlation hole”
    10.3. Analogy with a black hole: singular metric and Hawking temperature
11. Luttinger liquid of energy levels and DoS correlation at \beta=2,4
    11.1. Luttinger liquid from the Calogero-Sutherland model
    11.2. DoS correlation function for \beta=2,4
    11.3. Power-law banded random matrices as deformation of the Wigner-Dyson theory