

## Random Matrix Theory: Wigner-Dyson statistics and beyond

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1. Application of random matrices in physics
2. Invariant and non-invariant random matrix ensembles
  - 2.1. Probability distribution of invariant and Gaussian-non-invariant ensemble
  - 2.2. Strong and weak confinement
  - 2.3. Special cases of non-invariant ensembles: Rosenzweig-Porter and Power-law banded random matrices
3. Joint probability distribution of eigenvalues and eigenvectors of Wigner-Dyson ensemble and level repulsion
  - 3.1. The Jacobian and Vandermond determinant
  - 3.2. Level repulsion (poor men derivation)
4. Dyson symmetry classes and their extension
  - 4.1 Time-reversal symmetry and the Dyson symmetry classes
  - 4.2. Particle-hole symmetry and the 10-fold way Cartan symmetry classes
5. Level statistics of Wigner-Dyson RMT as 1D gas of quantum and classical particles
  - 5.1. Classical plasma of logarithmically-interacting particles
  - 5.2. Calogero-Sutherland models of interacting fermions
6. Wigner semi-circle and the probability of hole creation in a 1D log-interacting plasma
  - 6.1. Wigner semi-circle from the solution of integral equation for equilibrium profile of log-interacting 1D plasma
  - 6.2. Probability of a hole in log-interacting plasma and the tail of the level spacing distribution
  - 6.3. Hole production at weak confinement
7. Level compressibility, normalization sum rule and normalization anomaly
  - 7.1. Dos correlation function
  - 7.2. The level number variance
  - 7.3. The sum rule and the normalization anomaly
8. Solution of invariant ensembles by orthogonal polynomials
  - 8.1. Orthogonal polynomials and three-term recursive relation
  - 8.2. Expression for DoS correlation function in terms of orthogonal polynomials
  - 8.3. The Wigner-Dyson correlation kernel and higher-order correlation functions
9. WKB quasi-classical approximation for orthogonal polynomials and the one-dimensional Wigner crystal of energy levels
10. Logarithmic confinement and DoS correlation function
  - 10.1. Dos at logarithmic confinement
  - 10.2. Unfolding and the DoS correlation function: emergence of a “ghost correlation hole”
  - 10.3. Analogy with a black hole: singular metric and Hawking temperature
11. Luttinger liquid of energy levels and DoS correlation at  $\beta=2,4$ 
  - 11.1. Luttinger liquid from the Calogero-Sutherland model
  - 11.2. DoS correlation function for  $\beta=2,4$
  - 11.3. Power-law banded random matrices as deformation of the Wigner-Dyson theory

- 11.4. Levels statistics for the Power-Law banded random matrices as a Luttinger liquid at a finite temperature
- 12. Localization and multifractality
  - 12.1. What is multifractality of wave functions?
  - 12.2. Multifractal measures: moments of IPR and  $f(\alpha)$
- 13. Multifractal wave functions in Power-law banded random matrices
- 14. Multifractal wave functions in Rosenzweig-Porter random matrix theory
  - 14.1. Anderson localization and Mott's delocalization criteria for full random matrices
  - 14.2. Phase diagram for the Gaussian and log-normal Rosenzweig-Porter modelsc.
  - 14.3. Multifractal dimensions
- 15. Localization, diffusion and sub-diffusion in log-normal Rosenzweig-Porter model
  - 15.1. Return/survival probability
  - 15.2. Mini-bands
  - 15.3. Wigner-Weisskopf approximation and average return probability
  - 15.4. Stretch-exponential dynamics of return probability and sub-diffusion
  - 15.5. Dynamical phase diagram
- 16. Anderson model on random regular graph and the log-normal Rosenzweig-model
- 16.1. The Cayley tree and RRG
  - 16.2. Statistics of Green's functions on the Cayley tree
  - 16.3. Abou-Chakra-Thouless-Anderson (A-CTA) duality
  - 16.4. Symmetry of the moments of Green's functions
  - 16.5. Construction of the Rosenzweig-Porter model associated withRRG
  - 16.6. The log-normal Rosenzweig-Porter model with A-CTA symmetry