

## Basics of Green function techniques

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### 1. Introduction

- 1.1. Second quantization formalism: free bosons and fermions
- 1.2. Density matrix, Gibbs ensemble, Wick's theorem
- 1.3. Evolution operator, Liouville-von-Neumann equation

### 2. Basics of Keldysh technique

- 2.1. Keldysh Green functions for free particles (operator formalism)
- 2.2. Keldysh rotation, retarded, advanced and Keldysh Green functions. Equilibrium relation
- 2.3. Grassmanian algebra, path integral representation
- 2.4. Generating functional. Linear response theory and Kubo formula
- 2.5. Wigner transformation, semiclassical distribution function

### 3. Diffusive Fermi systems (diagrams)

- 3.1. White noise random potential as a model for weak impurities
- 3.2. Probability propagation, diffusion
- 3.3. Drude formula

### 4. Disordered Fermi systems (NLSM)

- 4.1. Hubbard-Stratanovich transformation, Q-matrix
- 4.2. Saddle point manifold, NLSM action
- 4.3. Inhomogeneous case: Usadel equation

### 5. Superconductivity

- 5.1. BCS model, Hubbard-Stratanovich transformation. Nambu space
- 5.2. Vicinity of critical temperature: Cooper instability, Ginzburg-Landau functional
- 5.3. Self-consistency equation below  $T_c$

### 6. Diffusive superconductors

- 6.1. NLSM for superconducting systems. Saddle point approximation, angular parametrization
- 6.2. Usadel equation. Boundary conditions (superconductor-insulator, superconductor-metal, tunneling contact)
- 6.3. Usadel equation out of equilibrium: the use of Keldysh technique

### 7. Applications to superconducting heterostructures

- 7.1. One-dimensional S-N-S junction: Density of States and tunneling conductivity
- 7.2. Two-dimensional S-N-S array: superconducting islands on graphene
- 7.3. Interplay of Coulomb anomaly and superconductivity
- 7.4. Superconductor-ferromagnet-superconductor junctions
- 7.5. Non-equilibrium effects: critical current of nanowire in presence of an injection